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ANALYSIS OF OPTIMIZATION METHODS FOR SOLVING THE PROBLEM OF DEVELOPING RATIONAL VARIANTS FOR EQUIPPING THE ARMED FORCES WITH AIRCRAFT

The article is dedicated to solving the topical scientific-practical problem of forming rational variants for the technical equipping (rearmament) of the armed forces with aircraft by optimizing the set of programmatic measures for maintaining their serviceability, modernization, and procurement. This problem is formalized as an integer linear programming problem of the "knapsack type" ("optimal loading"). A systemic analysis of optimization methods suitable for solving this problem of developing rational variants for equipping the armed forces with aircraft has been conducted. The objective function is defined as the maximization of the aviation's combat potential, subject to a number of constraints, primarily on financial expenditure. A comparative analysis of exact (brute force, branch and bound, dynamic programming) and heuristic (greedy, genetic) algorithms for solving the posed problem is conducted. Their advantages, disadvantages, computational complexity (performance), accuracy, and limitations on the number of variables are analyzed. The results of the analysis are systematized in the form of a comparative table with corresponding conclusions.

Keywords: *combat potential, armed forces aviation, aircraft, airworthiness maintenance, modernization, procurement, optimization, algorithm.*

Introduction

Problem Statement. The experience of modern military conflicts, particularly the full-scale russian aggression against Ukraine, irrefutably proves that air superiority is one of the key factors in achieving strategic advantage. In the context of ongoing hostilities, the issue of maintaining the required (specified) level of combat potential (CP) of the aviation component of the Armed Forces (AF) of Ukraine has become exceptionally acute.

From the standpoint of the systemic methodology of armament theory, AF aviation is considered as a specific armament system of an aviation grouping – a "triad" of interdependent components, which combines a system of combat assets, a system of command and control assets, and a system of combat support assets, and which is under the influence of external factors of a military-political, military-economic, military-technical, and operational-tactical nature [1].

An integral indicator that characterizes the quality of the aviation armament system as a complex organizational-technical system is its CP, which in the general case is a function of the military-technical capabilities of combat assets, command and control assets, and combat support assets [1].

At the core of the conceptual approach to forming the required outline of the AF aviation armament system lies the determination and comparison of the required (specified) and existing levels of CP, the identification of the CP deficit magnitude, and the substantiation of ways (measures) aimed at eliminating this deficit. That is, in other words – the optimization of the CP level, namely – determining its rational value, taking into account available resources – financial, economic, scientific-technical, production-technological, human, temporal, etc. In essence, this is a process aimed at forming the required outline of the prospective AF aviation armament system, specifically regarding its equipping, first and foremost, with combat assets – military aircraft.

The main ways to ensure the required (specified) level of CP for the AF aviation armament system are (Fig. 1):

– maintaining the airworthiness of aircraft currently in operation, primarily by ensuring the serviceability of aviation equipment through the timely (planned) execution of relevant programmatic measures (PM) – factory repair, transitioning to on-condition maintenance, and extending service life indicators;

– increasing the military-technical level of aircraft currently in operation through their modernization, which is aimed at expanding combat capabilities by ensuring multifunctionality, all-weather, and round-the-clock combat use, equipping them with high-precision air-launched weapons, modern navigation systems and onboard defense systems, modern avionics, etc.;

– rearmament (technical renewal) of the aviation fleet through the procurement of new, modern models of aircraft. In this case, aircraft of foreign and/or domestic (own) production may be procured. When procuring foreign-produced aircraft, the PM considered include aircraft created without the participation of Ukrainian enterprises and aircraft created with their participation (joint projects). When procuring domestically produced aircraft, the PM considered include aircraft created solely by Ukrainian enterprises, aircraft created with the participation of foreign companies, and/or aircraft produced under license.

– military-material (technical) assistance from partner countries of the Aviation Coalition. It is advisable to consider this path only during the period of active combat operations, as upon the conclusion of the "hot" phase of the confrontation, such assistance will obviously be curtailed or significantly limited.

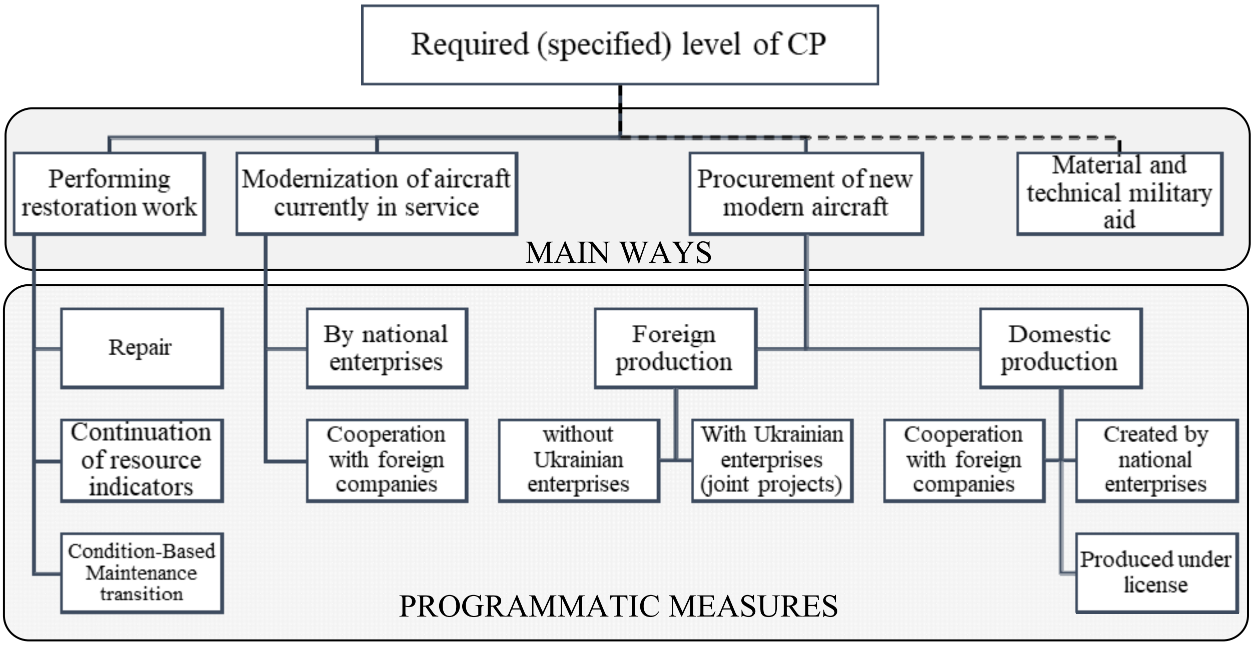

Fig. 1. Main ways of optimizing the combat potential level of the AF aviation armament system
Source: developed by the authors

Obviously, the formation of a rational variant of PM for equipping the AF with aircraft is a topical and complex scientific-practical problem, erroneous solutions to which carry risks of reducing the state's defense capability and inefficiently using the defense financial resources.

The purpose of the article is to present the results of scientific research regarding the systemic analysis of optimization methods that can be applied to solve the problem of forming rational variants for equipping the armed forces with aircraft – determining their advantages, disadvantages, and appropriate limits of application.

Analysis of Recent Research and Publications. The problem of scientifically substantiating the directions of development and selecting rational variants for equipping the armed forces with military equipment, including aviation technology (AT), is not new and has been studied by many domestic and foreign scientists.

The fundamental basis for its solution lies in the principles of systems theory, operations research, decision theory, and systemic analysis. This scientific problem area is reflected in the scientific works of professors B.O. Demidov, M.I. Lukhanin, M.M. Mitrakhovych, V.B. Tolubko, O.B. Leontiev, O.V. Samkov and others.

The basis for understanding and implementing the considered methods lies in the classic works on the theory of algorithms and data structures. Works [2, 3] describe in detail the basic approaches to algorithm development. The formalization of the posed problem as an integer linear programming task relies on the apparatus of operations research, thoroughly covered in [5, 6]. These works provide the mathematical toolkit for constructing guaranteed optimal solutions. As an alternative to exact methods for solving large-scale problems, genetic algorithms are considered. The theoretical foundations of their construction and application are set out in [7, 8].

However, despite the availability of a significant number of methods for solving individual tasks, the problem of their systemic analysis and the substantiation of a rational scope of application for forming variants of equipping with aircraft has not been fully resolved.

Main material

A PM shall be understood as a structurally separate, calendar-planned, and budgeted set of actions aimed at changing the state of the AT fleet (or its part) in order to achieve target values of combat potential, under given constraints, with an assessed feasibility (level of success in implementing the PM within the system of possible risks) and clear criteria for the result.

Forming rational variants for equipping the AF with aircraft shall be understood as determining, using optimization methods, such a ratio between repaired, modernized, and procured AC that ensures the maximum level of combat potential for AF aviation at the end of the planning period, within the limits of the available budget.

The formalized statement of the PM optimization problem is as follows:

$$CP(n_{11}, n_{12}, ..., n_{21}, n_{22}, ..., n_{ij}, ..., n_{IJ}) = \sum_{i=1}^{I}\sum_{j=1}^{J}(K_{ij}n_{ij})\Omega_{ij} \to \max \qquad (1)$$

subject to the conditions and constraints:

$$\sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}n_{ij} \le \overline{C} \qquad (2)$$

$$n_{ij} \ge 0,\ integer \qquad (3)$$

$$j = 1, 2, ..., J,\ J = 5 \qquad (4)$$

$$K_{ij} > 0;\ 0 < \Omega_{ij} \le 1;\ 0 < c_{ij} \le \overline{C} \qquad (5)$$

where CP – is the level of combat potential of a certain type of AF aviation; n_{ij} – the number of i-th type aircraft obtained through the implementation of the j-th PM; K_{ij} – the coefficient of the military-technical level of the i-th type aircraft, obtained through the implementation of the j-th PM; Ω_{ij} – the feasibility of the j-th PM for the i-th type aircraft; c_{ij} – the costs associated with the implementation of the j-th PM concerning the i-th type aircraft; $\overline{C}$ – allocated appropriations (designated cost limit).

In the problem statement (1) – (5), five main PMs are considered (J = 5):

– factory repair of existing domestic staff aircraft (j = 1);

– serial modernization of existing domestic staff aircraft (j = 2);

– serial modernization of existing foreign aircraft (j = 3);

– procurement of new foreign aircraft (j = 4);

– rent (leasing) of new foreign aircraft (j = 5).

The introduction of the PM feasibility indicator Ω_{ij} into the objective function (1) increases the accuracy of the CP indicator assessment by accounting for additional factors that directly influence its level, namely – the risks that may accompany the PMs for the repair, modernization, and procurement of aircraft].

The research problem in the formulation (1) – (5) belongs to the class of integer linear programming problems, specifically the "knapsack type" or "optimal loading" problem. The knapsack problem gets its name from the maximization task of packing the largest possible number of valuable (useful) items into a knapsack, given that the total volume or weight of all items that can fit into the knapsack is limited [2…4].

The classic "knapsack" problem is formulated as follows: from a given set of N i-th items, select a subset with the maximum total value (utility, cost) $\sum_{i=1}^{N} k_i x_i$ while simultaneously adhering to the constraint on their total weight $\sum_{i=1}^{N} v_i x_i \le W$. Where x_i is a Boolean variable that takes the value "1" if the i-th item is placed in the knapsack, and "0" if it is not.

There are several variations of the knapsack problem, the differences between which lie in the conditions imposed on the knapsack, the items, or their selection [3, 4]:

– classic problem (0-1 knapsack) – each item can be taken only once;

– bounded knapsack – each item can be taken a specific number of times;

– unbounded knapsack – each item can be taken an unlimited number of times;

– continuous (fractional) knapsack – any fractional part of an item can be taken;

– multiple-choice knapsack – there are several classes of items, from which one representative can be taken, and some items have a higher priority than others;

– multiple knapsack – there are several knapsacks with different capacities;

– multidimensional knapsack – there is more than one constraint on the knapsack.

The research problem in the formulation (1) – (5) belongs to the bounded multidimensional knapsack problem. In this case, the items are the PMs, the value k_i (utility) of an item is the coefficient of the military-technical level of the aircraft (K_{ij}), the weight V_i of an item is the costs (c_{ij}) associated with implementing the j-th PM, and the weight constraint W (knapsack capacity) is the limit of allocated funds for the implementation of the planned PMs ($\overline{C}$).

It should be noted that the knapsack problem belongs to the so-called NP-complete problems, which cannot be solved in polynomial time. The challenge in solving such problems is the absence of "efficient" (polynomial) algorithms that can solve them within an acceptable time. An algorithm is called "efficient" (polynomial) if its labor intensity and required machine memory volume depend polynomially on the problem's dimensionality, i.e., on the length of the binary record of all input data. In other words, an algorithm is called polynomial if its complexity $O(N)$ in the worst case is bounded above by some polynomial of N [3, 4].

When solving NP-complete problems, one has to choose between exact, but slow and resource-intensive algorithms, and approximate (heuristic), but fast algorithms.

An analysis of scientific publications, particularly [2…8], showed that the most widely used exact algorithms in practice are the methods of brute force (complete search), branch and bound, and dynamic programming, while among the approximate (heuristic) algorithms, the greedy and genetic algorithms are the most common.

An analysis of the possibility of applying these methods to solve the PM optimization problem for equipping AF aviation, as formulated in (1) – (5), showed the following.

Brute force (complete search) method. This is considered the most accurate method for solving "knapsack" type problems [5, 6]. To obtain a solution, it is necessary to go through all possible loading variants (PM variants). A simple recursive implementation of this approach is obvious, but the time complexity of this algorithm is $O(N!)$. That is, the algorithm has factorial complexity and can only work with small values of N. As N increases, the number of variants grows very quickly, and the problem becomes practically unsolvable by the brute force method.

Thus, since the brute force method is very labor-intensive, the size of the knapsack and the number of items must be limited when using it. Therefore, this method cannot be applied to unbounded and continuous knapsacks. When solving multiple and bounded knapsack problems by brute force, the number of items should not exceed 10 for a solution to be found [5, 9].

Advantages of the brute force method: provides an exact solution; simplicity of machine implementation.

Disadvantages of the brute force method: limited number of items ($N < 10$); low performance – the time complexity of the method is $O(N!)$.

Branch and bound method. In essence, this is an improved search method that excludes solutions known to be non-optimal. The method's algorithm ensures the construction of a decision tree and the pruning of branches (variants) known to be incorrect. Common to all branch and bound algorithms are the concepts of branching and calculating bounds. The pruning of variants occurs as follows: if a certain item (programmatic measure) causes the knapsack to be overloaded (exceeds the limit of allocated funds), then that branch of the decision is excluded from further consideration, because the knapsack will be overloaded (insufficient funds). This pruning accelerates the solving of the problem. However, it is not always possible to prune a sufficient number of variants to noticeably increase the operational speed. As practice shows, it is always possible to select input data for which the branch and bound method will yield a time estimate identical to that of a brute force search [3].

An obvious disadvantage of the branch and bound method algorithm when solving large-dimensional problems is the need to go through too many variants before the optimal one is found. However, thanks to the pruning of non-optimal solutions, the branch and bound method algorithm can independently limit an infinite solution space. Therefore, it can be applied to the unbounded knapsack, but cannot be applied to the continuous knapsack. For the bounded knapsack, the number of items should not exceed 20 [3].

The branch and bound method is irregular, meaning its specific implementations (algorithms) are tied to rules for partitioning into subsets (branching rules) and rules for constructing estimates of the objective function values on these subsets (constructing bounds), which depend on the specifics of the particular problems [5, 6].

Advantages of the branch and bound method: provides an exact solution; universal and simple to implement on a computer; possible significant reduction in problem-solving time.

Disadvantages of the branch and bound method: limited number of items ($N < 20$); in the worst case, it works like the brute force method.

Dynamic programming method. At the core of the method lies Bellman's principle of optimality: whatever the state of the system may be before the next step, one must choose the control at this step so that the gain at this step, plus the optimal gain on all subsequent steps, is optimal. That is, the optimal solution at a certain step is calculated based on optimal solutions found earlier (at previous steps). It follows from this that in order to find the optimal solution at the final step, one must first find the optimal solution for the first, then for the second, and so on, until all steps up to the last one have been processed.

The dynamic programming method for the knapsack problem provides an exact solution, and it simultaneously computes solutions for all knapsack sizes from 1 to *max W*. However, storing the table of values and remembering whether each item was taken or not requires on the order of $O(N \times max\ W)$ of machine memory; the time complexity of the method is also $O(N \times max\ W)$ [5, 6].

Advantages of the dynamic programming method: provides an exact solution; high operational speed compared to other algorithms (for $N < 50$); optimal knapsack loads are calculated for all capacity values from 1 to *max W*.

Disadvantages of the dynamic programming method: the requirement for integer values of item weights (cost of PM implementation); low performance for a large number of items ($N > 50$) – time complexity is $O(N \times max\ W)$.

Greedy algorithm. Solving the knapsack problem using the greedy algorithm consists of the following steps:

– the set of items (programmatic measures) N is sorted in descending order of the items' specific value (cost per unit of the aircraft's military-technical level), i.e., such that:

$$\frac{k_1}{v_1} \geq \frac{k_2}{v_2} \geq \ldots \geq \frac{k_N}{v_N} \qquad (6)$$

– starting from an empty set, items (programmatic measures) from the sorted set N are sequentially added to the approximate solution (which is initially empty);

– with each subsequent addition of an item (programmatic measure) to the knapsack (to the PM variant), a check is performed to ensure its weight (cost of PM implementation) does not exceed the remaining knapsack capacity (remaining allocated funds);

– the process of constructing the approximate solution to the knapsack problem is complete once all items (programmatic measures) have been reviewed.

Advantages of the greedy algorithm: works with practically any value of N; simple to implement on a computer; not demanding on machine memory volume; high performance – the time complexity of the method is $O(N \times log\ N)$; provides an optimal solution for the continuous knapsack problem.

Disadvantages of the greedy algorithm: does not guarantee finding the optimal solution; it is a single-criterion algorithm, therefore it cannot be applied to the multidimensional knapsack.

Genetic algorithm. This algorithm is based on the evolutionary principles of heredity, variability, and natural selection. The general scheme of the genetic algorithm is presented in Fig. 2 [7, 8].

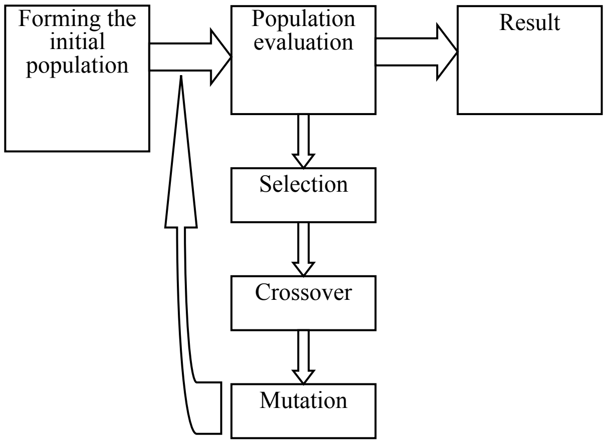

Fig. 2. The general scheme of the genetic algorithm
Source: developed by the authors based on [7, 8]

The genetic algorithm operates on a population of individuals (PM variants), where the chromosome (indicators of the programmatic measure) of each individual encodes a possible solution to the problem (ensuring maximum CP within the allocated funds).

The genetic algorithm is one of the fastest algorithms. However, a limitation of this algorithm is that its chromosomes are encoded discretely, meaning it cannot be used to solve continuous and infinite knapsack problems. Furthermore, the genetic algorithm does not guarantee finding the optimal solution. This occurs because the algorithm may terminate not only when the optimal solution is reached, but also under the following conditions: the maximum specified number of iterations is completed; the allotted time for the algorithm has expired; or there are no significant changes when transitioning to a new generation [7].

Advantages of the genetic algorithm: high performance (high operational speed).

Disadvantages of the genetic algorithm: does not guarantee finding the optimal solution (finds a local extremum instead of a global one); cannot be used to solve continuous and infinite problems.

To select a specific method for solving the research problem (1) – (5), an analysis of the approaches discussed above was conducted based on the following indicators: solution accuracy; algorithm operational speed; number of variables accounted for; and simplicity of implementation on a computer.

The results of the analysis of the approaches considered above for solving the integer linearprogramming problem of the “knapsack” type are presented in Table 1.

Table 1

Results of the analysis of methods (algorithms) for solving the "knapsack" type integer linear programming problem

Method (algorithm) name	Indicators for evaluating methods (algorithms)			
	Solution accuracy	Complexity (performance/speed)	Number of variables accounted for	Simplicity of implementation on a computer
Brute force (complete search)	100 %	O(N!) (low)	< 10	YES
Branch and bound	100 %	≤ O(N!) (depends on the specifics of the task)	< 20	YES
Dynamic programming	100 %	O(N × max W) (acceptable)	< 50	YES
Greedy algorithm	73 %	O(N × log$N$) (high)	Unbounded	YES
Genetic algorithm	89 %	– (is set a priori)	Unbounded	NO

Source: developed by the authors based on [2…8]

In practice, the choice between methods is made depending on the specifics of the particular task at hand – the required solution accuracy, the number of variables (items, elements, etc.), the speed at which the solution is obtained, the labor costs for the method's software implementation, and so on. For example, if it is necessary to calculate an absolutely optimal solution, regardless of the time spent (within reasonable limits), then exact methods must be used. But if accuracy can be sacrificed and the number of elements is large, then heuristic methods must be applied.

Conclusions

The research results presented in the article on optimization methods for solving the problem of forming rational variants for equipping the AF with aircraft show that the choice of a specific method depends on the balance between the required solution accuracy, the dimensionality of the problem (number of variables), and the available computational resources.

The most accurate methods for solving it are brute force (complete search), branch and bound, and dynamic programming; however, their use is limited by the number of variables and significant computational costs. The greedy and genetic algorithms provide sufficient operational speed and acceptable result accuracy for large-dimensional problems, which makes them suitable for preliminary or rapid assessment of programmatic measures for aviation technology modernization.

When choosing a method for solving the PM optimization problem as formulated in (1) – (5), two main requirements were considered: the result must provide an exact optimal solution, given that the number of variables does not exceed 20; the software implementation of the method must be relatively simple. The branch and bound method meets these requirements.

The obtained results can be used as a methodological basis for building decision support systems in the field of planning the development of aviation technology for the state aviation of Ukraine, as well as in forming substantiated modernization programs within the framework of long-term defense capability planning.

Direction of further research. Further research should be directed toward evaluating the possibility of combining optimization methods to increase the accuracy and speed of solving the problem, and toward the software implementation of the algorithm as a module (component) of a management decision support system.

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АНАЛІЗ МЕТОДІВ ОПТИМІЗАЦІЇ ДЛЯ ВИРІШЕННЯ ЗАДАЧІ ФОРМУВАННЯ РАЦІОНАЛЬНИХ ВАРІАНТІВ ОСНАЩЕННЯ ЗБРОЙНИХ СИЛ ЛІТАЛЬНИМИ АПАРАТАМИ

О.Є. Мавренков, О.В. Карпенко

Досвід сучасних збройних конфліктів, зокрема повномасштабної агресії російської федерації проти України, беззаперечно доводить, що панування в повітрі є одним з ключових факторів досягнення стратегічної переваги. В умовах триваючих бойових дій питання підтримання потрібного (заданого) рівня бойового потенціалу авіаційної компоненти Збройних Сил України набуло виняткової гостроти.

Статтю присвячено вирішенню актуальної науково-практичної задачі формування раціональних варіантів технічного оснащення (переозброєння) збройних сил літальними апаратами шляхом оптимізації комплексу програмних заходів з підтримання їх справності, модернізації та закупівлі. Зазначену задачу формалізовано у вигляді цілочисленої задачі лінійного програмування “рюкзачного типу” (“оптимального завантаження”), проведено системний аналіз методів оптимізації, придатних для розв’язання даної задачі формування раціональних варіантів оснащення збройних сил літальними апаратами. Цільовою функцією визначено максимізацію бойового потенціалу авіації при дотриманні низки обмежень, насамперед на витрату фінансів. Проведено порівняльний аналіз точних (повного перебору, гілок і границь, динамічного програмування) та евристичних (жадібного, генетичного) алгоритмів для вирішення поставленої задачі, проаналізовано їхні переваги, недоліки, обчислювальну складність (швидкодію), точність та обмеження за кількістю змінних. Результати аналізу систематизовано у вигляді порівняльної таблиці з відповідними висновками.

Ключові слова: бойовий потенціал, авіація збройних сил, літальний апарат, підтримання льотної придатності, модернізація, закупівля, оптимізація, алгоритм.